

Nuclear Matter with Quark-Meson Coupling II: Modeling a Soliton Liquid

Nir Barnea* and Timothy S. Walhout[†]

ECT, European Centre for Theoretical Studies in Nuclear Physics and Related Areas,
Strada delle Tabarelle 286, I-38050 Villazzano (Trento), Italy*

and

Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Trento

February 9, 2008

Abstract

We study in further detail a nontopological soliton model with coupling between quarks and mesons selected as promising in a previous study that employed the so-called Wigner-Seitz approximation for dense systems. Here we go beyond this approximation by introducing the disorder necessary to reproduce the liquid state, using the significant structure theory of Jhon and Eyring. We study nuclear matter, with particular interest in the transition to a quark plasma. The model studied is a variation of the chromodielectric model of Fai, Perry and Wilets, where explicit coupling to a scalar meson field is introduced.

PACS: 24.85.+p, 12.39.Ki, 21.65.+f

Keywords: Nuclear matter, Chromodielectric model, Quark-gluon plasma, Solitons

*e-mail: barnea@ect.it

[†]e-mail: walhout@ect.it

1 Introduction

This is the second of two papers that seek to select and develop a soliton model of nucleons for application to dense matter and, in particular, the transition to deconfined quark matter. In the previous paper, hereafter referred to as (I), we presented the motivation for our choice of a particular class of nontopological soliton models, the Friedberg-Lee type models [1], which have explicit quark degrees of freedom and a dynamical confinement mechanism resulting from a composite scalar gluon field that forms a solitonic bag in which constituent quarks then reside. We considered extensions of these models that include explicit meson degrees of freedom coupled linearly to the quarks in order to obtain a reasonable description of nuclear interactions. For simplicity, we have studied — and here also study — only models that include scalar and vector mesons, neglecting in these initial investigations any possible explicit effects of pions in dense matter (although the scalar meson can be considered an effective two-pion resonance). We avoid any double counting of hadronic degrees of freedom by including only nuclear constituent quarks in our calculations. Gluons enter our calculations only through the scalar glueball field; perturbative gluonic effects are ignored in nuclear matter.

The distinguishing feature between the various models we considered in (I) is the precise form of the coupling between the quarks and the glueball field. We compared the various FL models by studying dense matter within the Wigner-Seitz approximation. In general, one can find parameter choices that produce reasonable results for free nucleon properties independent of the particular form of the quark-glueball coupling. It is in dense matter, where quark bags begin to touch, that the various models are distinguished. We found that models which have a quark-glueball coupling in accord with the dictates of the chiral chromodielectric model (χ CD) show a behavior more in line with phenomenology. In these models, the quark-gluon coupling is of leading order two or greater in the glueball field, which is essential for the elimination of transitions to unphysical quark plasma phases at unrealistically low densities. Furthermore, it was found that coupling the quarks to a scalar meson field ensures saturation. The quark-meson coupling is taken to be independent of the glueball field within the mean field approximation to avoid unphysical transitions in dense matter, as was detailed in (I).

Here we wish to further study the model selected in (I) by going beyond the relatively rough approximations used there in modeling the liquid state. The Wigner-Seitz approximation consists in assuming that each nucleon is confined by interactions with its nearest neighbors to a given volume, equal to the inverse of the baryon density, known as the Wigner-Seitz cell. For a solid cubic lattice, for example, the Wigner-Seitz cell is a cube. Here, we are interested not in the solid but rather the liquid state, and so the usual choice is to take the Wigner-Seitz cell to be a sphere in the hope that one thereby better models the disorder of a liquid. This, however, is clearly not enough: if we look at models of the liquid state used by physical chemists in order to describe molecular liquids (models that passed out of use several decades ago after the development of large-scale computers enabling the use of molecular dynamics and Monte Carlo techniques), the Wigner-Seitz approximation

employed in (I) corresponds to the cell model of Lennard-Jones and Devonshire [7], which does much better at reproducing the solid state than the liquid state [8]. Instead, we shall employ a refinement of the cell model — namely, the Significant Structure Theory of Jhon and Eyring [9] — which introduces holes into the system in order to account for the disorder present in liquids.

As a matter of fact, in (I) we assumed that the Wigner-Seitz cell is simply a sort of “average snapshot” of the nuclear medium felt by the quarks inside an otherwise freely moving nucleon. Thus we proceeded by subtracting away energy due to spurious center of mass motion (due to the fact that the nucleon was not constructed by putting the quarks in a good momentum state), and then took the kinetic energy of the system to be that of a free Fermi gas. Clearly, this approximation can only be justified at low densities. As the density increases the motion of an individual nucleon is affected by the medium, and this leads us to consider the Wigner-Seitz cell not just as a boundary upon the quark wave functions that build up a nucleon, but also as a restriction upon the motion of the nucleon itself. This leads to the considerations of the previous paragraph, which shall be further developed in Sec. 3.

The outline of the paper is as follows. The nontopological soliton model used is reviewed in Sec. 2. In Sec. 3 we discuss various attempts to model a soliton liquid, then motivate and introduce the particular model based on significant liquid structures that we shall use here. In Sec. 4 we present the resulting equations of state for nuclear matter and discuss the transition to quark matter. A general summary and discussion of the two papers is given in Sec. 5.

2 The Model

The nontopological soliton model we study here is based upon the chiral chromodielectric model of Fai, Perry and Wilets [2]. In its full version, the model contains quark and gluon degrees of freedom. A scalar glueball field σ couples to the quarks, and colored gluons A_μ^a are treated perturbatively. The scalar field provides absolute confinement of both quarks and gluons and gives constituents a mass. Meson exchange is surely present in this model, but for simplicity we alter the original χ CD by dropping the gluon field A_μ^a and ignoring sea quarks. Instead, as in quark-meson coupling models, we introduce a scalar meson ϕ . The vector meson V_μ , which provides repulsion in quantum hadrodynamics, is not necessary here since the soliton structure provides repulsion between nucleons, and so for simplicity we set $V_\mu = 0$. We assume the scalar meson couples linearly to the quarks and take the quark-meson vertex to be independent of σ . The Lagrangian density for our model is

$$\begin{aligned} \mathcal{L} = & \bar{\psi} [i\gamma^\mu \partial_\mu - g(\sigma) - g_s \phi] \psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) \\ & + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_s^2 \phi^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \end{aligned} \quad (1)$$

where

$$U(\sigma) = \frac{a}{2!} \sigma^2 + \frac{b}{3!} \sigma^3 + \frac{c}{4!} \sigma^4 + B. \quad (2)$$

We have set the current quark masses to zero. The parameters are as chosen in (I). The constants of the potential are $a = 50\text{fm}^{-2}$, $b = -1300\text{fm}^{-1}$ and $c = 10^4$, so that $U(\sigma)$ has a local minimum at $\sigma = 0$ and a global minimum at $\sigma = \sigma_v = 0.285\text{fm}^{-1}$, the vacuum value. The mass of the glueball excitation associated with the σ field is $m_{GB} = \sqrt{U''(\sigma_v)} = 1.82\text{GeV}$ and the bag constant is $B = 46.6\text{MeV/fm}^3$. The quark- σ coupling is

$$g(\sigma) = g_\sigma \sigma_v \left[\frac{1}{\kappa(\sigma)} - 1 \right], \quad (3)$$

where we choose the chromodielectric function $\kappa(\sigma)$ to be

$$\kappa(\sigma) = 1 + \theta(x)x^3[3x - 4 + \kappa_v] \ ; \ x = \sigma/\sigma_v \ , \quad (4)$$

In the following we take $g_\sigma = 3$ and $\kappa_v = .1$.

We solve the Euler-Lagrange equations in the mean field approximation, replacing the glueball field σ by the classical soliton solution $\sigma(\vec{r})$ and the meson field by its expectation value in the nuclear medium $\langle \phi \rangle = \phi_0$. The resulting equations for the quark and the scalar soliton field are solved in a Wigner-Seitz cell of radius R by implementing boundary conditions based upon Bloch's theorem, as detailed in (I). The quark spinor in the lowest band is assumed to be an s -state

$$\psi_k = \begin{pmatrix} u_k(r) \\ i\sigma \cdot \hat{r} v_k(r) \end{pmatrix} \chi, \quad (5)$$

and we make the simplifying assumption of identifying the bottom of the lowest band by the demand that the derivative of the upper component of the Dirac function disappears at R , and the top of that band by the demand that the value of the upper component is zero at R [5]. The resulting equations for the spinor components are

$$\frac{du_k}{dr} + [g(\sigma) - g_s\phi_0 + \epsilon_k] v_k = 0 \quad (6)$$

$$\frac{dv_k}{dr} + \frac{2v_k}{r} + [g(\sigma) - g_s\phi_0 - \epsilon_k] u_k = 0 \ . \quad (7)$$

The equation for the soliton field is

$$-\nabla^2\sigma + U'(\sigma) + g'(\sigma)\rho_s(r) = 0. \quad (8)$$

The quark density ρ_q and the quark scalar density ρ_s are given by

$$\rho_q(r) = \frac{n_q}{4\pi\bar{k}^3/3} \int_0^{\bar{k}} d^3k \ [u_k^2(r) + v_k^2(r)] \ , \quad (9)$$

$$\rho_s(r) = \frac{n_q}{4\pi\bar{k}^3/3} \int_0^{\bar{k}} d^3k \ [u_k^2(r) - v_k^2(r)] \ , \quad (10)$$

where the band is filled up to \bar{k} . The quark functions are normalized to unity in the Wigner-Seitz cell. The boundary conditions for the soliton field are $\sigma'(0) = \sigma'(R) = 0$. The boundary

conditions for the quark functions at the origin are given by $u(0) = u_0$ and $v(0) = 0$, where u_0 is determined by the normalization condition

$$\int_0^R 4\pi r^2 dr (u(r)^2 + v(r)^2) = 1 . \quad (11)$$

The boundary conditions at $r = R$ are given by $u'_b(R) = 0$ and $v_b(R) = 0$ for the bottom of the lowest band, and $u_t(R) = 0$ for the top of this band. Using these equations we can solve for the corresponding ϵ_b and ϵ_t . We assume the tight-binding dispersion relation $\epsilon_k = \epsilon_b^2 + (\epsilon_t - \epsilon_b) \sin^2(\pi s/2)$, with $s = k/k_t$, and that the band is filled right to the top — that is, $\bar{k} = k_t$. With this dispersion relation and filling, the nucleon energy is given by

$$E_N = 3n_q \int_0^1 ds s^2 \left\{ \epsilon_b + (\epsilon_t - \epsilon_b) \sin^2 \left(\frac{\pi s}{2} \right) \right\} + \int_0^R 4\pi r^2 dr \left[\frac{1}{2} \sigma'(r)^2 + U(\sigma) \right] . \quad (12)$$

In order to correct for the spurious center of mass motion in the Wigner-Seitz cell the nucleon mass at rest is taken to be

$$M_N = \sqrt{E_N^2 - \langle P_{cm}^2 \rangle_{WS}} , \quad (13)$$

where $\langle P_{cm}^2 \rangle_{WS} = n_q \langle p_q^2 \rangle_{WS}$ is the sum of the expectation values of the squares of the momenta of the $n_q=3$ quarks.

At low density the band width vanishes and the quarks are confined in separate bags. Then we can assume the individual nucleons move around as a gas of fermions with effective mass M_N given by Eq. (13), so the nucleon energy is $E_N^{(g)} = \sqrt{M_N^2 + k^2}$. The total energy density at nuclear density ρ_B is thus

$$\mathcal{E}_g = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d\vec{k} \sqrt{M_N^2 + k^2} + \frac{1}{2} m_s^2 \phi_0^2, \quad (14)$$

where $\gamma = 4$ is the spin-isospin degeneracy of the nucleons. The Fermi momentum of the nucleons is related to the baryon density through the relation

$$\rho_B = \frac{\gamma}{6\pi^2} k_F^3 = \frac{3}{4\pi R^3} . \quad (15)$$

The total energy per baryon is given by $E_g = \mathcal{E}_g/\rho_B$. We have used the label g to indicate that these expressions correspond to a gas-like phase. The constant scalar meson field ϕ_0 is determined by the thermodynamic demand of minimizing \mathcal{E} :

$$\frac{\partial \mathcal{E}_g}{\partial \phi_0} = 0 . \quad (16)$$

Our mean field equations are similar to those of quantum hadrodynamics, the difference here being that the nucleon now has structure and thus the meson field couples to the nucleon through its quarks.

Let us review the treatment of dense matter in (I). We started with the so-called Wigner-Seitz approximation, which is often used in soliton calculations, since it is the simplest picture of dense matter available. In this approximation, each soliton is confined to a unit

cell, which we can view from two different perspectives. If we choose periodic conditions at the cell boundary, we have the usual Bloch approach to the solid state: the quarks play the role of the electrons and the scalar glueball field takes the role of the ions in the usual crystal formulation. If instead we want to model the liquid state, we need to somehow introduce some disorder into the system. From this view, then, we take the Wigner-Seitz approximation as a sort of averaging over the rest of the system: for the purposes of constructing an individual nucleon, we ignore the motion of the nucleons, instead adding “by hand” the kinetic energy of a free gas to the system. This is justified to the extent that each nucleon’s motion describes slow degrees of freedom, whereas the constituents are fast degrees of freedom that react essentially instantaneously to changes in the relative arrangement of the nucleons — that is, if the Born-Oppenheimer approximation is valid.

Thus in (I) we calculated the equation of state of nuclear matter within the second scheme. The mass of the nucleon was calculated as a function of the radius of the (spherical) Wigner-Seitz cell, for which it was then necessary to subtract away spurious center-of-mass motion. This was done using approximate relations discussed in detail in [6]. Note that within the first scheme, the quarks are not assumed to be in a state of good momentum, and this kinetic energy is not spurious.

3 Significant Structure Theory

In (I) we used the Wigner-Seitz approximation to determine the effective nucleon mass in nuclear matter, but then gave the nucleons the kinetic energy of a Fermi gas. In this approximation, the quarks feel the nuclear medium, but the nucleons themselves do not. That is, we assume the quarks adjust instantly to the medium, forming $3q$ collective states that move relatively slowly and essentially freely through the medium. The interactions between nucleons occur only indirectly through the effective mass and the mesonic mean field. Clearly, such an approximation cannot be accurate at high densities, when the finite size of the nucleon becomes important, for nucleons will not then move freely. Instead, we need to model the liquid state, where any individual nucleon will range about the system over long time scales, but will be localized on shorter time scales. One would still like to approach the problem using the Wigner-Seitz approximation, insisting now that the nucleon also feels the medium and does not leave the cell. Clearly, this is a very restrictive assumption, and physical chemists long ago understood that this corresponds more closely to the solid than the liquid state. Nevertheless, this will provide a starting point for the model of the liquid state we shall adopt in the following, so let us pursue this approach further.

Ideally, we would like to allow our nucleon to rattle about in its Wigner-Seitz cell in order to extract a potential. This would entail dropping the assumptions of spherical symmetry for the bag and the use of only s -wave quark states. Instead, we shall attempt to model the nucleon’s motion at high density as follows. First, we assume that the motion is harmonic — that is, the center of mass \mathbf{R}_{cm} of the nucleon is never far from the center of the WS cell

$\mathbf{R}_0 = 0$. Now, the average of a harmonic potential in the ground state is

$$\langle V \rangle = \left\langle \frac{1}{2} M_N \omega_N^2 \mathbf{R}_{cm}^2 \right\rangle = \frac{3}{4} \omega_N, \quad (17)$$

from which we find the natural frequency ω_N as a function of the effective nucleon mass M_N and the mean square of the center of mass coordinate. Next, we identify \mathbf{R}_{cm} as the center of the quark distribution in the cell, ignoring thereby any motion of the soliton bag σ . (Viewing Fig. (3) of (I), we see that the soliton bag begins to be “squeezed” by the WS boundary at $R \approx 1.5\text{fm}$, so that at least for densities higher than this we might argue that the bag is essentially fixed.) Taking \mathbf{R}_{cm} to be the center of mass coordinate of three quarks, we have

$$\langle \mathbf{R}_{cm}^2 \rangle = \left\langle \left[\frac{1}{3} (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \right]^2 \right\rangle = \frac{1}{3} \langle r_q^2 \rangle_{WS}. \quad (18)$$

Now we identify the last average with the mean square charge radius of the nucleon

$$\begin{aligned} \langle r^2 \rangle_{WS} &= \frac{\int_0^R d^3r r^2 \rho_q(r)}{\int_0^R d^3r \rho_q(r)} \\ &= \frac{1}{\frac{4}{3}\pi k^3} \int_0^{\bar{k}} d^3k \frac{\int_0^R d^3r r^2 [u_k^2(r) + v_k^2(r)]}{\int_0^R d^3r [u_k^2(r) + v_k^2(r)]}. \end{aligned} \quad (19)$$

This is clearly only a rough estimate of the center of mass motion of the nucleon. The nucleon energy in the solid is now

$$E_N^{(s)} = M_N + \frac{3}{2} \omega_N, \quad \text{with} \quad \omega_N = \frac{3}{2M_N \langle R_{cm}^2 \rangle}. \quad (20)$$

This approximation corresponds to subtracting away spurious kinetic energy from the soliton energy Eq. (12), namely,

$$\begin{aligned} E_{sp} = E_N - E_N^{(s)} &= \sqrt{M_N^2 + \langle P_{cm}^2 \rangle_{WS}} - M_N - \frac{3}{2} \omega \\ &\approx \frac{1}{2M_N} \left(\langle P_{cm}^2 \rangle_{WS} - \frac{9}{\langle 2R_{cm}^2 \rangle_{WS}} \right). \end{aligned} \quad (21)$$

Approximating $\langle R_{cm}^2 \rangle$ by $\frac{1}{3} \langle r^2 \rangle_{WS}$ can only be valid at high density. At low density, this surely breaks down for then the quarks cannot reach the WS boundary unless the bag itself is allowed to move. Moreover, the present approximation corresponds to treating the soliton matter as an Einstein solid (we have implicitly averaged over the Bloch momenta \mathbf{K} corresponding to the lattice of nucleons: a more accurate treatment of the solid would put the three quarks in a state of good $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$ and find a frequency $\omega_N(\mathbf{K})$ that is a function of the Bloch momentum). The total energy density in this “solid” phase is

$$\mathcal{E}_s = \rho_B \left(M_N + \frac{3}{2} \omega_N \right) + \frac{1}{2} m_s^2 \phi_0^2. \quad (22)$$

The constant scalar meson field ϕ_0 is again determined by the thermodynamic demand of minimizing \mathcal{E} . Both M_N and ω depend implicitly upon ϕ_0 , so that this equation must be solved iteratively in conjunction with those for σ and ψ .

This gives the equation of state for solid nuclear matter, which is not of much interest in itself. However, this is useful for building a model of the liquid state at high density based upon significant structure theory [9]. The essential idea is to isolate those configurations that make the significant contribution to the partition function. Based upon experimental observations of molecular liquids, the liquid state is viewed as a close-packed lattice with holes present that destroy any long-range order. The volume of the system increases by increasing the number of holes, with the volume of each hole equal to the average volume occupied by a close-packed molecule, since this balances the competing demands for greater entropy and lower energy. A molecule neighboring a hole can move into the vacancy, creating a new hole at the site it left. Each hole thus replaces three vibrational with three translational degrees of freedom. Thus the liquid is represented as a combination of molecules with solid-like properties (those next to filled sites) and gas-like properties (those next to vacant sites).

Now consider this model of the liquid state applied to dense solitonic matter. (Such an application has been studied previously for dense skyrmion matter in Ref. [21].) First, we note that our assumptions of a spherical Wigner-Seitz cell, which reflects a sort of average over nearest neighbor positions, and the Einstein approximation that ω_N is independent of the Bloch momentum \mathbf{K} , which ignores long-range correlated vibrations of the nucleons — approximations that would be rather severe if we truly wished to model a solid — are instead appropriate for the present application. We need only add the holes to ensure the disorder corresponding to a liquid state. So let V_l be the total volume of the liquid and v be the volume of each cell (occupied or unoccupied). If there are N nucleons and N_h holes, then $V_s = Nv$ is the total volume of the occupied cells and $V_g = V_l - V_s = N_h v$ is the total volume of the holes. On average, a nucleon will encounter a neighbor on a fraction $\frac{N}{N+N_h} = \frac{V_s}{V_l}$ of its trips and a hole on a fraction $1 - \frac{V_s}{V_l} = \frac{V_g}{V_l}$ of its trips. Thus there are $3N \frac{V_s}{V_l}$ solid-like and $3N \frac{V_g}{V_l}$ gas-like degrees of freedom, and the liquid partition function is

$$Z_l = Z_s^{N \frac{V_s}{V_l}} Z_g^{N(1 - \frac{V_s}{V_l})}. \quad (23)$$

This results in the following nucleon energy in the liquid state:

$$E_N^{(l)} = \frac{V_s}{V_l} E_N^{(s)}(v) + \frac{V_l - V_s}{V_l} \tilde{E}_N^{(g)}(n_g), \quad (24)$$

where $n_g = \frac{N_g}{V_g} = \frac{N}{V_l}$ is the “density” of the gas-like part of the system, which from the last equality is equal to the baryon density ρ_B . The average solid-like nucleon energy $E_N^{(s)}(v)$ is given by Eq. (20), with M_N and ω_N depending upon v through the Wigner-Seitz radius $R = (3v/4\pi)^{1/3}$. The average gas-like nucleon energy is instead

$$\tilde{E}_N^{(g)}(n_g) = \frac{3\gamma}{4\pi k_g^3} \int_0^{k_g} d^3k \sqrt{M_N^2(v_l) + k^2}, \quad (25)$$

with $k_g = (6\pi^2 n_g / \gamma)^{1/3} = (6\pi^2 \rho_B / \gamma)^{1/3}$ and $v_l = V_l / N = 1/n_g$. The total energy density of the system in the liquid phase is then

$$\mathcal{E}_l(v) = \rho_B^2 v E_N^{(s)}(v) + \rho_B (1 - \rho_B v) \tilde{E}_N^{(g)}(\rho_B) + \frac{1}{2} m_s^2 \phi_0^2 - \frac{1}{2} m_V^2 V_0^2. \quad (26)$$

Once again, ϕ_0 is determined by minimizing \mathcal{E}_l , and so for nonzero g_s we must solve anew the equations for σ and ψ consistent with the mean field value $\phi_0(v, \rho_B)$ determined from this new liquid equation of state. In Eq. (26), the cell volume v is to be taken as a parameter. Note that the WS cell volume is no longer $1/\rho_B$ when holes are present. Instead, we can define a new radius $R_l = (3/4\pi\rho_B)^{1/3}$ which is half the average spacing between nucleons in the liquid state.

There is some question as to whether one should take the effective nucleon mass in (25) to be $M_N(v)$ or $M_N(v_l) = M_N(\rho_B^{-1})$. Since a single nucleon can have both gas-like and solid-like degrees of freedom, it might be more consistent to use the former choice in calculating the gas-like part of the nucleon energy. However, there seems little reason to insist that the inertial masses corresponding to the solid-like harmonic motion and the gas-like translational motion are the same. Moreover, the latter choice guarantees that our liquid EOS reduces to the Fermi gas EOS given by (14) at low density. Thus our equation of state (26) interpolates smoothly between high-density and low-density pictures of the liquid state.

4 Results

4.1 Nuclear matter

In Figs. 1-3 we show the energy per baryon of χ CD solitonic nuclear matter for three different values of the quark-meson coupling. In each graph we display the “gas” equation of state (14) and a set of significant liquid structure curves characterized by various choices of the WS cell volume $v = 4\pi R^3$ in (26). In addition, the upper curve labelled “solid” in these figures is given by (14) with the condition $n_g = 1/v$ — or, equivalently, $R_l = R$ — which implies that the number of holes is zero, so that the nuclear motion is entirely solid-like. Note that our approximations in estimating ω_N break down at low density. In particular, the “solid” curve is not to be trusted above $R \approx 1.5\text{fm}$: here, the bag is no longer squeezed by the WS boundary and its motion probably can no longer be ignored.

As $v \rightarrow 0$, the significant structure curve approaches the “gas” curve, which certainly underestimates the energy per nucleon. Without empirical evidence of a transition from liquid to solid nuclear matter, we are unable to fix v directly. (Some models predict such a transition; however, it is more widely believed that the transition to the quark-gluon plasma will preclude a transition to solid nuclear matter.) Here, we can just treat v as a parameter in choosing the EOS that best fits the empirical data at the saturation point. We find that, for the parameter sets we studied, the curves that best fit the empirical EOS for nuclear matter are given by the values $g_s = 2, R = 0.4\text{fm}$ and $g_s = 3, R = 0.7\text{fm}$. The values of the Fermi momentum, binding energy, and compression modulus at the saturation point are $k_s = 1.06\text{fm}^{-1}$, $E_s = 22\text{MeV}$ and $K = 1082\text{MeV}$ for the former, and $k_s = 0.95\text{fm}^{-1}$, $E_s = 24\text{MeV}$ and $K = 671\text{MeV}$ for the latter curve. These are to be compared with the empirical values $k_s = 1.36\text{fm}^{-1}$, $E_s = 16\text{MeV}$ and $K \approx 200\text{MeV}$. In particular, we see a significant improvement in the value of the compression modulus with respect to the calculation of (I), although it is still rather high with respect to empirical values.

4.2 Quark matter

In the high density limit, the preferred phase in the nontopological soliton models we are studying is a uniform plasma characterized by the solution $\sigma = 0$. That is, the soliton bags disappear, and one is left with a quark gas. The energy density of the quark plasma is

$$\mathcal{E}_q = \frac{3k_F^4}{2\pi^2} + B, \quad (27)$$

where the Fermi momentum is related to the baryon density ρ_B as $k_F = (6\pi^2\rho_B/\gamma)^{1/3}$ — the degeneracy of the quark gas is $n_q\gamma$ and the quark density is $n_q=3$ times the baryon density. The bag constant is $B = 46.6\text{MeV}/fm^3$ for our choice of parameters. In contrast to our assumption about the solitonic phase, the energy density of the quark gas is altered significantly by perturbative gluonic contributions, especially at higher densities. In this phase, the χ CD model in the mean field approximation is equivalent to perturbative QCD. As shown in [22], for example, adding the lowest order gluonic corrections then gives

$$\mathcal{E}_q = \frac{3k_F^4}{2\pi^2} \left\{ 1 + \frac{2\alpha_s}{3\pi} + \frac{\alpha_s^3}{3\pi^2} \left[6.79 + 2 \ln \left(\frac{2\alpha_s}{\pi} \right) \right] \right\} + B, \quad (28)$$

where the leading-logarithm expression for the strong coupling constant is

$$\alpha_s(k_F) = \frac{6\pi}{29 \ln(k_F/\Lambda)}. \quad (29)$$

We take the QCD scale parameter to be $\Lambda \sim 180\text{-}200\text{MeV}$.

In Fig. 4 we show the two nuclear matter curves selected above along with the quark-gluon plasma EOS given here. In addition, we show an “empirical” nuclear matter EOS given by

$$E_B \approx \frac{K}{18} \left(\frac{k_F^3}{k_s^3} - 1 \right)^2 + M_N^{(as)} - E_s, \quad (30)$$

with $M_N^{(as)} - E_s = 1160\text{MeV}$ and $K = 200\text{MeV}$. (The bag constant B appearing in the quark-gluon plasma energy has been set in fitting the free soliton mass and rms radius, and thus for purposes of comparison we take the low density limit of the “empirical” curve to coincide with the free soliton mass.) Note that the $g_s = 2$ shown here has a transition to the solid state at $k_F = (9\pi)^{1/3}/2R = 3.8\text{fm}^{-1}$ and the $g_s = 3$ at 2.2fm^{-1} , both well past the transition points to the quark-gluon plasma. Clearly, the saturation points of the model curves occur at densities that are too low. Nevertheless, the very fact that we find qualitative agreement with the empirical nuclear matter EOS is quite encouraging.

5 Conclusions and Outlook

In this paper we have developed an improved modeling of the liquid state of solitonic matter based upon the significant structure model used in physical chemistry. We have applied this

to the study of nuclear matter within a nontopological soliton model with explicit quark degrees of freedom. In particular, initial studies in (I) indicated that among several such similar models, the chiral chromodielectric model gave results most in line with empirical expectations. When a scalar meson is allowed to couple to the quarks, saturation can be achieved. The calculations in (I) were based upon a modeling of nuclear matter that assumes the solitons move essentially freely through the medium. This clearly will break down at and above nuclear saturation density, where the size of the individual solitons and the spacing between solitons in the medium become comparable. The significant structure model of the liquid state used in the present paper, however, is designed for densities near the transition to the solid state. We have thus developed here an equation of state that interpolates between models designed for low and high densities. It is encouraging to find that without changing the free nucleon properties we can adjust the quark-meson coupling to reproduce the saturation point of nuclear matter found in (I). Indeed, we find that the compression modulus is improved significantly when calculated using the significant liquid structure model.

With the χ CD model we are able to treat the transition to the quark-gluon plasma consistently. This is the point of developing this model: confinement is dynamical. Thus the parameters governing the nuclear and plasma phases are in principle the same. Clearly, the results we have found here, while qualitatively in agreement with empirical estimates, are not quantitatively useful. In particular, the saturation point of nuclear matter occurs at far too low a density for the parameters used in our calculation. Moreover, the mass of the free nucleon is too high. We have not tried too hard to improve our results by adjusting the parameters of the model. Probably one can find a better choice of parameters than those for which we have done the calculations here, but we must emphasize that there seems to be a limit in how well one can do. This can be seen already in studying the free nucleon. There, we are unable to find a parameter set that reproduces the nucleon mass and rms radius exactly while still giving reasonable values for the glueball and bag constant. We compromised here by fitting the nucleon rms radius well while keeping the glueball mass and bag constant in their accepted ranges. This resulted in a free nucleon mass that was too high.

In trying to reproduce the empirical saturation point of nuclear matter, however, we found a general scaling phenomenon. Keeping the saturation energy roughly correct, it seems difficult to change the parameters in such a way as to get the correct density expect by a rough overall scaling of all predicted quantities. Thus getting the correct saturation density entails lowering the rms radius along with a corresponding increase in the mass of the nucleon. This is suggestive. As a matter of fact, the quark-meson model we developed here is better suited for nuclear matter than for isolated nucleons. This is because an isolated nucleon will surround itself with a pion cloud, whereas (unless pion condensation occurs) the effects of pions in nuclear matter are likely to be small if we already have scalar mesons. Thus we can consider the model used here as actually better used for describing the quark core of the nucleon. The addition of the pion will allow the quark core to shrink and lower the energy of the free nucleon with respect to that of the quarks. With the shrinking of the quark core, one can expect a corresponding decrease in the volume per nucleon at the saturation point.

Thus we view the addition of pions as an essential improvement to our model. This, of course, was always obvious: we certainly cannot hope to reproduce the long-range part of the nucleon-nucleon interaction without pions. Our results indicate that the presence of pions is necessary in order to reproduce the structure of the nucleon as well. This comes as little surprise. There are of course other improvements that can be made upon our calculations, such as a better handling of the Bloch boundary conditions and quark wave functions [4] and an improved treatment of the corrections due to spurious center of mass motion. Perturbative corrections due to gluons and mesons about the mean field should eventually be considered as well.

Having said (that is, written) this about improving the model, we should not lose sight of the original goal of our work. What we wanted to do first of all was see if we could distinguish among the various nontopological soliton models on the market by studying dense matter. To this we can answer that the chiral chromodielectric model appears to be more in line with empirical expectations. Then, we wanted to develop as simple a model as possible that could give a rough qualitative fit to both free nucleon and dense nuclear matter properties, thus providing a reasonable starting point for more sophisticated models that can provide truly quantitative predictions. To this we can also answer that the chiral chromodielectric model, when modified according to the local uniform approximation by the addition of a scalar meson field, would appear to be such a model. In fact, this model not only gives a good qualitative and rough quantitative fit to the empirical equation of state, it also predicts an increase of the nuclear rms radius in the nuclear medium, a result that is in accord with the EMC effect. Moreover, the results presented here seem to indicate that with the addition of pions we would have good possibilities of obtaining a quantitatively accurate fit to single nucleon properties and to the empirical nuclear matter equation of state. This would provide a reliable estimate of the bag constant and therefore a consistent and accurate treatment of the transition from nuclear to quark-gluon matter as a true transition between two phases of a single model.

References

- [1] R. Friedberg and T. D. Lee, *Phys. Rev.* **D 15** (1977) 1694.
- [2] Fai, Perry and Wilets, *Phys. Lett.* **B 208** (1988) 1.
- [3] G. Krein, P. Tang and A.G. Williams, *Nucl. Phys.* **A 523** (1991) 548.
- [4] U. Weber and J. McGovern, *Phys. Rev.* **C57** (1998) 3376; nucl-th/9710021, (1997).
- [5] M. C. Birse, J. J. Rehr and L. Wilets, *Phys. Rev.* **C 38** (1988) 359.
- [6] L. Wilets, “Nontopological Solitons,” World Scientific Lecture Notes in Physics Vol. 24, World Scientific (1989).
- [7] J.E. Lennard-Jones and A.F. Devonshire, *Proc. Roy. Soc.* **A163** (1937) 53.
- [8] J. A. Barker, “Lattice Theories of the Liquid State,” (Pergamon, London, 1963).
- [9] H. Eyring and M. S. Jhon, “Significant Liquid Structures,” (Wiley, New York, 1969); M.S. Jhon and H. Eyring, in: “Theoretical Chemistry: Advances and Perspectives,” vol. 8A (Academic Press, New York, 1977).
- [10] J. D. Walecka, *Ann. Phys. (N.Y.)* **83** (1974) 491.
- [11] B. D. Serot and J. D. Walecka, *Advan. Nucl. Phys.* vol. 16 (1986) 1.
- [12] P. A. M. Guichon, *Phys. Lett.* **B 200** (1988) 235.
- [13] S. Fleck, W. Bentz, K. Shimizu and K. Yazaki, *Nucl. Phys.* **A 510** (1990) 731.
- [14] H. Muller and B. K. Jennings, nucl-th/9706049, (1997).
- [15] X. Jin and B. K. Jennings, *Phys. Rev.* **C 55** (1997) 1567.
- [16] P. A. M. Guichon, K. Saito and A. W. Thomas, *Nucl. Phys.* **A 601** (1996) 349.
- [17] P. G. Blunden and G. A. Miller, *Phys. Rev.* **C 54** (1996) 359.
- [18] H. B. Nielsen and A. Patkos, *Nucl. Phys.* **B 195** (1982) 137.
- [19] M. K. Banerjee and J. A. Tjon, *Phys. Rev* **C 56** (1997) 497.
- [20] W. Koepf, L. Wilets, S. Pepin and Fl. Stancu, *Phys. Rev* **C 50** (1994) 614.
- [21] T.S. Walhout, *Nucl. Phys.* bf A519 (1990) 816.
- [22] B.A. Freedman and L.D. McLerran, *Phys. Rev.* **D16** (1977) 1130, 1169.

Figure 1: Energy per baryon as a function of average liquid radius $R_l = (3/4\pi\rho_B)^{1/3}$ for various values of the Wigner-Seitz radius $R = (3v/4\pi)^{1/3}$ for quark-meson coupling $g_s = 1$. Also shown are the curves for the “gas” limit $R \rightarrow 0$ and the “solid” limit $R_l = R$.

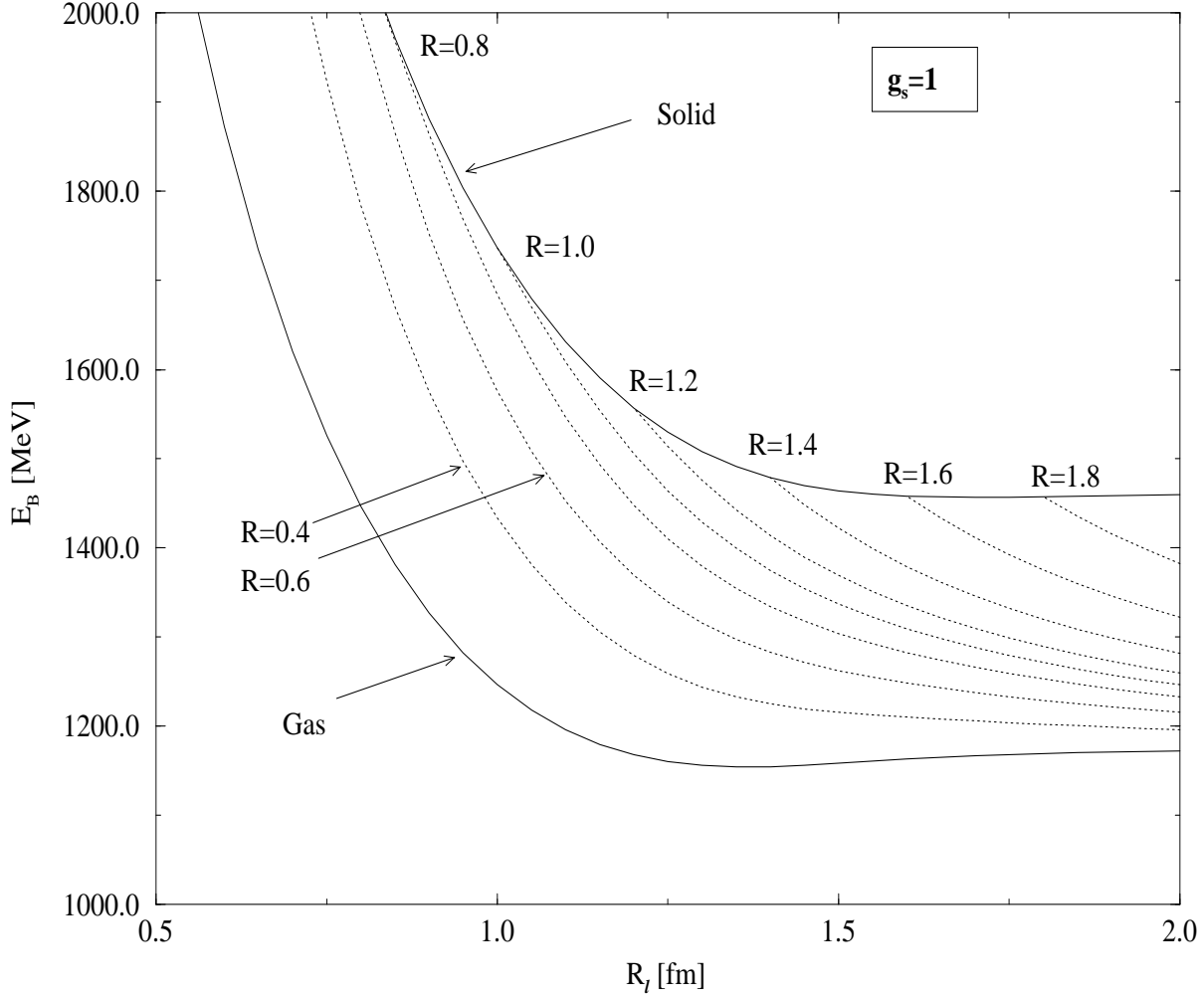


Figure 2: As in Fig. 1, for the choice of quark-meson coupling $g_s = 2$.

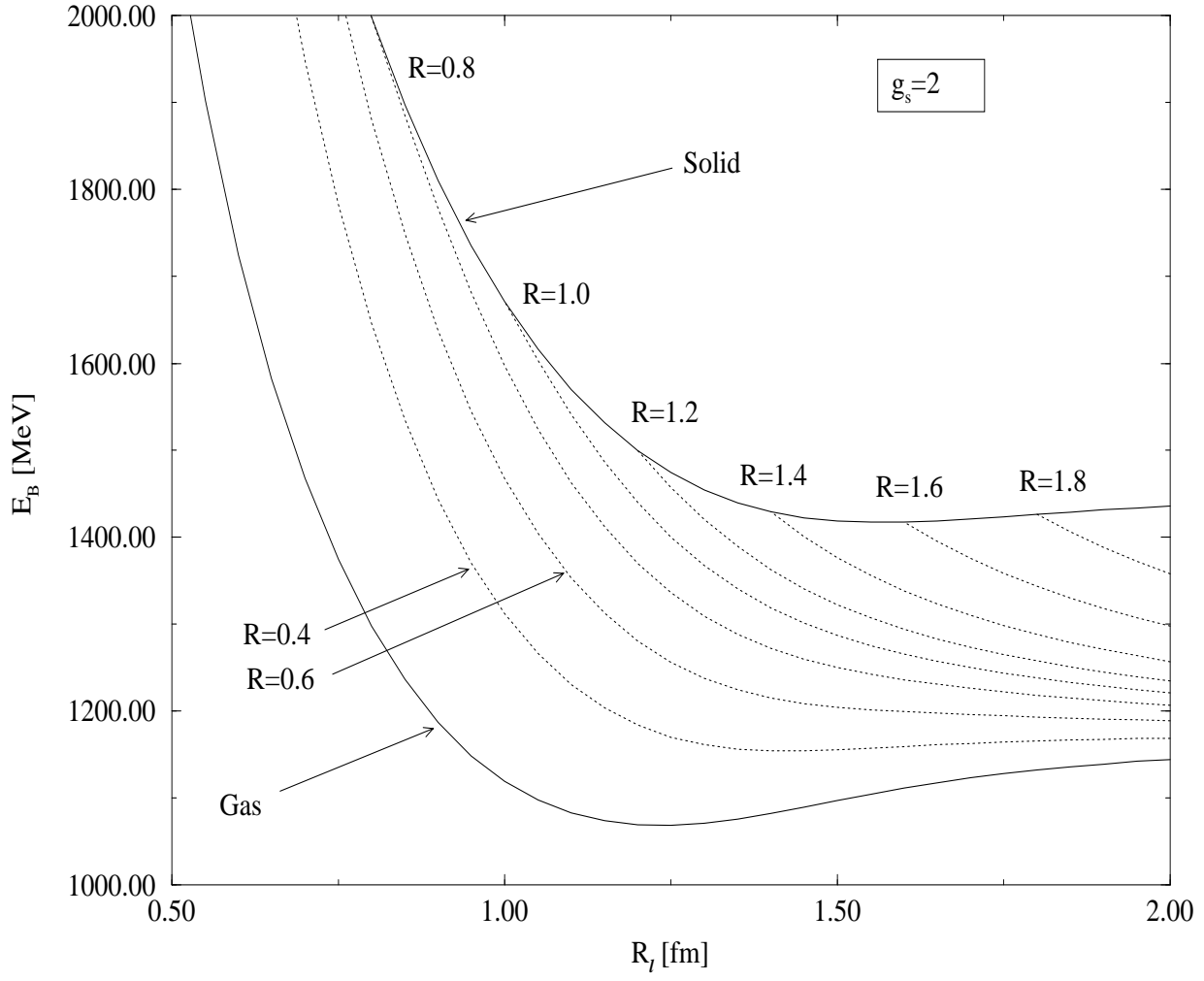


Figure 3: As in Figs. 1 and 2, but with $g_s = 3$.

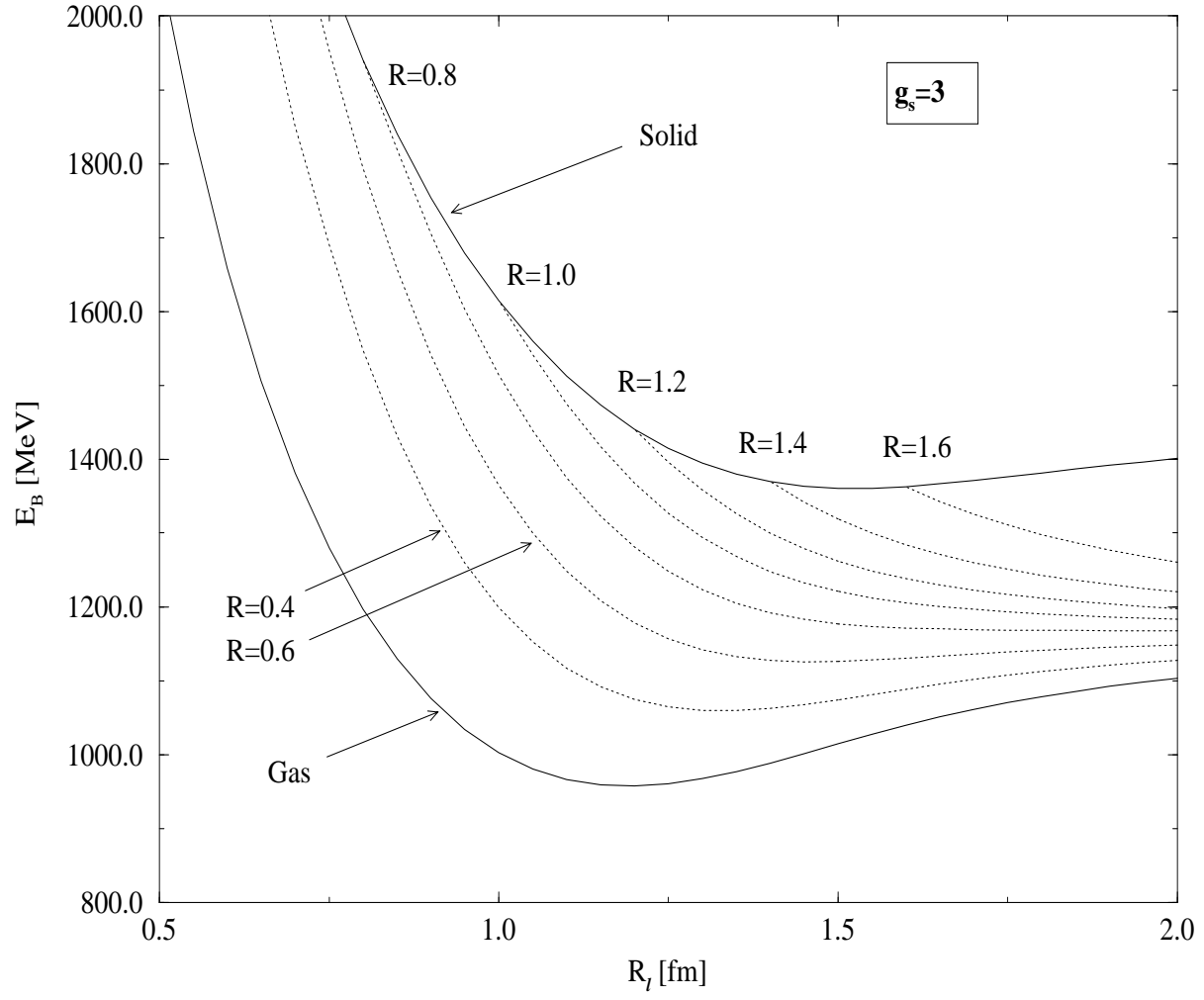


Figure 4: Quark-gluon plasma (QGP) and nuclear matter energy per baryon number as function of the Fermi momentum. The curves are as described in the text.

